through the use of resonant modes having $E_n=0$ at dielectric surfaces adjacent to metal walls.

The precision of the two methods has been evaluated by allowing f, D, and L to vary in the formulas. For $D\approx 0.3$ inch and $L\approx 0.1$ inch, tolerances of ± 0.1 percent in f, and ± 0.0005 inch in D and L lead to maximum possible errors of about ± 0.5 percent in ϵ_r, and probable errors of about ± 0.2 percent. An improvement of accuracy by at least a factor of 10 is feasible, but is not necessary for most practical purposes.

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The Measurement of Phase at UHF and Microwave Frequencies

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Abstract—**A theoretical analysis and a unifying classification of methods of measuring phase at UHF and microwave frequencies are presented. The coherent phase bridge circuits are analyzed in terms of the type of modulation applied to the channels of the bridge and the type of combiner and mixer employed at the output of the bridge. In this analysis and classification, identifying characteristics, and some of the relative advantages and disadvantages of these circuits become obvious.**

I. Introduction

Within the past twenty years there have been many systems and techniques proposed for the measurement of phase in the UHF and microwave range of frequencies. In two recent papers Sparks presented a review and comparison of several of these systems and an excellent bibliography [1], [2]. In general, however, the literature is specialized and widely scattered.

Our purpose here is to present a theoretical analysis of bridge circuits suitable for the measurement of phase at these UHF and microwave frequencies. This analysis will lead to a classification for these circuits which clearly delineates the relationships between them. In an attempt to provide a unifying treatment the literature is extensively referenced.

The analysis is intended to be limited to circuits suitable for use at UHF frequencies and above, although it will be obvious that with present day components many of them can be used at lower frequencies. These circuits or systems are concerned with the measurement of phase of CW signals or signals with controlled repetitive modulation. Specialized techniques, for example such as those that might be required for unknown or uncooperative pulse modulation, are not considered.

II. Classification of Measurement Systems

The measurement of the relative phase of an RF signal involves either a comparison between the phase of an unknown signal and that of a reference signal, or a more fundamental measurement that involves the measurement of the changing character of the unknown signal with time. We are here concerned primarily with the former and will identify systems suitable for this comparison as belonging to one of several classes or subclasses.

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Nearly all of the available phase measurement techniques in the UHF and microwave range of frequencies make use of a bridge circuit which has two or more channels as shown in simplified form in Fig. 1(a). The unknown or changing phase of a signal under test is compared with that of a coherent reference signal by combining and mixing these signals in a nonlinear element. Because this is a heterodyne and basically a linear mixing process, the relative phase shift between these two signals is preserved in the resultant output signal from the mixer. The character of the output signal depends upon the type of modulation applied to the signals in the two channels and the type of combiner and mixer employed at the output of the bridge.

The physical form of the bridge and the component under test may take many forms. As shown in simplified form in Fig. 2, the measurable quantity, the argument of the complex transmission or reflection coefficient may, for example, be the phase shift through a circuit element, a sheet of dielectric, a radome, or a plasma, or the relative phase of radiated fields, scattered fields, or coupled fields as a function of distance or angular orientation. The form shown for scattered fields is the familiar reflection coefficient bridge. While these forms may present different physical problems and possibly a different interpretation of the measurements, as far as processing the signals to obtain phase information is concerned, they are all a common measurement problem.

We can divide the measurement systems into two basic classes. The first class (Class I) includes those in which the RF test and reference signals are directly mixed and compared. The second class (Class II) includes those in which these two signals are both translated to a lower intermediate frequency before they are compared.

Within Class I we can identify three types: a) those in which the signals in both channels have identical modulation; b) those in which the signal in neither one of the channels is modulated; and c) those in which one or the other of the channels is modulated. This latter case includes those systems that could be devised which have a basic difference in the modulation applied to each channel. The first and last type will each have a distinctive output for the four common forms of amplitude modulation, double sideband with carrier present (DSBWC), double sideband with suppressed carrier (DSBSC), single sideband with carrier present (SSBWC), and single sideband with suppressed carrier (SSBSC). Types a) and b) include those commonly identified as the phase bridge and the interferometer systems, although these two labels fit equally well to all of the systems. Type c) includes among others those commonly identified as the modulated subcarrier, homodyne, and serrodyne systems.

These latter systems in which only one of the channels is modulated have been identified as coherent or synchronous detection systems; however, all the Class I systems are coherent superhetrodyne systems in which the test signal usually takes the place of the carrier and the reference signal which is furnished by the source of the signal under test usually takes the place of the local oscillator signal. They are thus zero frequency IF systems [3]–[5].

In a zero frequency IF system the desired output from the mixer is a dc signal or in a modulated system a signal at the modulation frequency. The frequency of modulation can be tuning fork or crystal controlled and hence very narrow bandwidth amplifiers with their desirable frequency and noise discriminating characteristics can be used after the mixer. The coherent detection system under proper conditions can achieve the sensitivity and

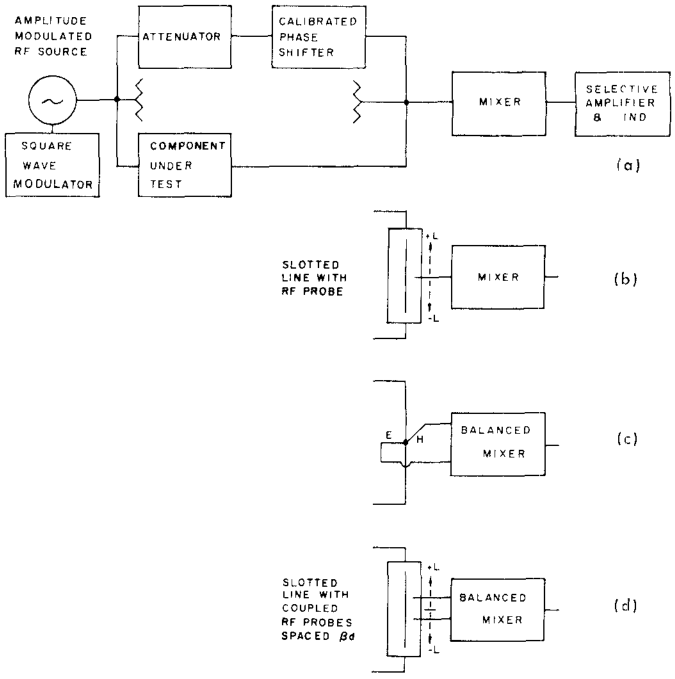


Fig. 1. Basic phase bridge with alternate forms of power combiners and mixers.

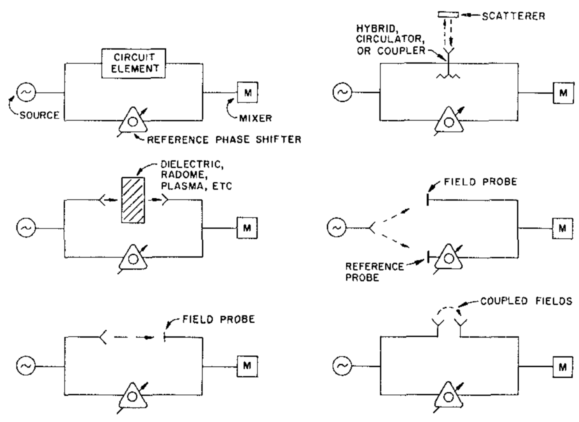


Fig. 2. Symbolic form of the phase bridge for various types of phase measurement.

the linear response of mixer operation over an extremely wide dynamic range.

In the analysis of these systems we will consider the signals propagating in the two arms and incident upon the power combiner to be of the form

$$e_1 = E_1(1 + m_1 \cos \omega_{m_1}t) \cos (\omega_1 t + \theta) \quad (1)$$

$$e_2 = E_2(1 + m_2 \cos \omega_{m_2}t) \cos (\omega_2 t + \phi) \quad (2)$$

where m is the amplitude modulation index, ω_m is the angular modulating frequency, and $\omega = \omega_1 = \omega_2$ is the angular frequency and θ and ϕ the relative time phase of the signals e_1 and e_2. This representation assumes sinusoidal modulation of e_1 and e_2. In actual practice the modulating voltage may be a square wave. However, since it is assumed that the mixer will be followed by a narrow-band filter and amplifier, this square wave may be decomposed into its Fourier components and only the fundamental components considered.

We will consider low level mixing and assume that the transfer function of the mixer element is of the form

$$i = a_1 e + a_2 e^2 + \cdots + a_n e^n + \cdots \quad \text{where } e = e_1 \pm e_2. \quad (3)$$

For purposes of classification and comparison of the various systems it will be adequate to consider only terms through the second order in the expansion. Although there are higher order terms present, the character of the output of the system is determined by the form of the first few terms [5]–[6]. It should be recognized that the comparison that we are to make of these systems may change if very large signals are impressed upon the mixer element and the transfer function can no longer be described by (3). We will first consider the Class I systems.

III. Class I-A: Coherent Phase Bridge with Both Channels Modulated

A. *With a Single-Ended Mixer*

The basic phase bridge shown in Fig. 1(a) has probably been used more than any other system. The signal from an amplitude modulated source is divided into a reference channel and a test channel where it is modified. These signals in the two channels are then recombined and applied to a single-ended mixer element.[1] The output from this mixer is fed to a selective amplifier with a pass band centered at the modulation frequency ω_m and an indicator. Since the selective amplifier will pass only components of the signal that vary at the modulation frequency ω_m the output of the selective amplifier will be

$$V \cong Km[E_1^2 + E_2^2 \pm 2E_1E_2 \cos (\phi - \theta)] \cos \omega_m t \quad (4)$$

where K is a constant and $m = m_1 = m_2$. The plus or minus sign will depend upon whether the output of the power combiner is the sum or difference of the signals in the two channels. A simple tee may be used as a combiner but because considerable isolation is required between the two channels a directional coupler or hybrid is preferred [8]. The last term in this expression is a linear term that is a product of the magnitudes of the two signals and the cosine of the relative phase difference between them. There are, however, two nonlinear terms which depend upon the magnitude of the two signals. If we assume that the output of the combiner is the sum of the signals, V can be made zero if

$$E_1^2 + E_2^2 + 2E_1E_2 \cos (\phi - \theta) = 0 \quad (5)$$

which is possible if $E_1 = E_2$ and $\phi - \theta = n\pi$, where $n = 1, 3, 5, \cdots$. The bridge can thus be balanced to a null by adjustment of the calibrated reference phase shifter, provided e_1 is equal in amplitude to e_2. A change in the phase of the unknown can be compensated for by a change in the calibrated phase shifter and hence the relative phase of the unknown signal can be determined.

If $E_1 \neq E_2$, V can still be minimized by adjustment of the phase shifter but the minimum becomes shallow and difficult to determine for ratios of E_1/E_2 greater than 6 or 8 dB. Nevertheless, because of its simplicity this system has been widely used; for example by Redheffer [9], Samuel [10], Iams [11], Hines and Boehnker [12], Beam et al. [13], Morita [14], King [15], Ajoika [16], Monteath et al. [17], and Zacharias [18]. It was refined by Magid [19] who took extreme care to isolate the two channels. Using a precision calibrated phase shift standard, based upon a directional coupler and precision sliding short, he obtained an estimated measurement accuracy of better than 0.3 degree for test and reference signals of equal amplitude. Beatty [20] proposed a complex impedance meter based upon this circuit and Magid's phase shifter, which is capable of high accuracy. This basic system is, however, useable only over a relatively small dynamic range of the amplitude of the unknown or test signal and without this extreme care in construction and use only moderately accurate.

In an alternate form of the bridge the two channels are combined by connecting them to opposite ends of a slotted line as shown in Fig. 1(b). This form is often identified as an interferometer because of the interference pattern or standing wave created in the slotted line by the two signals. This standing wave may be sampled by a moveable probe and movement of the probe corresponds to the simultaneous addition of phase shift to one channel and subtraction of this same phase shift from the other. Thus the output from the selective amplifier is

$$V \cong K_1 m[E_1^2 + E_2^2 + 2E_1E_2 \cos(\phi \pm 2\beta L - \theta)] \cos \omega_m t \quad (6)$$

where K_1 is a constant which is dependent upon the probe coupling. This output can be nulled if $E_1 = E_2$ and $\phi - \theta = n\pi \mp 2\beta L$ where n = 1, 3, 5, $\cdots$. Thus, movement of the probe a distance L to rebalance the bridge indicates a shift in the phase of the unknown signal by $2\beta L$ radians. Since the slotted line functions as a phase shifter and combiner, the calibrated phase shifter may

[1] The general problem of the mixing of two modulated waves by an ideal linear rectifier has been treated in detail by Aiken [7].

not be necessary unless it is desired to position the minima of the standing wave at a particular place on the slotted line. This system is extremely simple but because of the nonlinear terms in the output it suffers from the disadvantanges of the first system. In addition, the conventional slotted line does not provide isolation between the channels. Attenuators of at least 10 dB, and preferably 20 dB, or ferrite isolators must be included in each channel adjacent to the slotted line and to the power divider to minimize reflections and interaction between the channels.

Because of its simplicity and the fact that the components are usually readily available, this system has found wide use. Ring in 1948 used a slotted line as a combination power divider and phase shifter [21]. Among others, Share [22] has discussed the system shown in Fig. 1(b) and commercial phase bridges which use this circuit are available. This type of system must be considered only of very moderate precision with a possibly attainable resolution of ± 0.5 degree and accuracy of ± 1.5 or 2 degrees using a vernier scale on the slotted line.

B. With a Balanced Mixer

The basic phase bridge can be improved by combining the signals in a balanced mixer [23]. If the test and reference signals given by (1) and (2) are connected to the side arms of a magic tee or a coaxial hybrid,[2] as shown in Fig. 1(c) and in Fig. 3, the output from the other two ports of the hybrid will be the sum and the difference of these two inputs. If these outputs are applied to matched mixers and subtracted in a well balanced transformer or difference circuit, the output from the selective amplifier at angular frequency ω_m will be

$$V \cong 4K_2 m[E_1 E_2 \cos(\phi - \theta)] \cos \omega_m t \tag{7}$$

where K_2 is constant and $(\phi-\theta)$ is the difference in phase between the test and reference signals. Note that the E_1^2 and E_2^2 terms in (4) have been cancelled and the output is equal to a constant times the product of E_1, E_2, and the cosine of the phase angle between e_1 and e_2. Thus a null in V can be obtained that is independent of the relative amplitudes of E_1 and E_2 if

$$\cos(\phi - \theta) = 0 \tag{8}$$

or $\phi-\theta=\pm n\pi/2$ where $n=1, 3, 5, \cdots$.

This relationship depends upon m being small. Although V departs from a pure cosine variation for larger values of m, the essential features are preserved, in that V is still zero-valued at and reverses sign on either side of $\phi-\theta=\pm n\pi/2$. Thus the output has the characteristics of a phase discriminator.

[2] A waveguide or coaxial hybrid is here considered to be a four-port device that divides a signal incident upon a given port into equal amplitude, in-phase or out-of-phase, components at the two adjacent ports. The short slot couplers, and 3 dB coaxial couplers that are sometimes called hybrids may be used; however the outputs at adjacent ports of these devices differ in phase by only 90°.

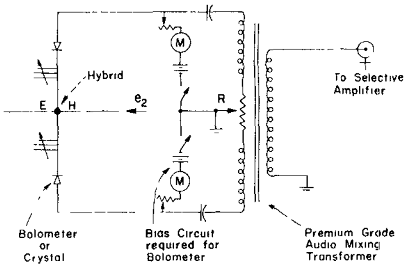


Fig. 3. Balanced mixer for modulation frequencies in the audio range.

Pound [24] originally used the hybrid as a microwave phase discriminator for a klystron stabilization circuit. This phase discriminator circuit was used to produce an error signal for a servo driven variable phase shifter in an automatic phase measurement system by Hines [25] and in an automatic impedance recorder by Gabriel [26]. The advantages of this balanced mixer as a phase discriminator were also pointed out by Morita and Sheingold [27], Richmond [28], Slocum and Augustine [29], Kofoid [30], Bengt-Olof Ås [31], Scharfman [32], and others. Commercial phase measuring instruments which are based upon the use of an accurately calibrated delay line as a reference and combination of the signals in a balanced mixer are available [33].

As was true for the systems using an unbalanced mixer, there must be adequate isolation between channels and the input reflection coefficient of all components must be small. This can be assured by using directional couplers or hybrids for power dividing and combining, tuners on each side of the unknown, isolators if available, and well-matched attenuators as pads.

Equation (7) indicates that the output voltage V, if plotted as a function of the phase difference between e_1 and e_2, is ideally a sinusoid with zero crossovers corresponding to $\phi-\theta=n\pi/2$ and an amplitude which is a function of the relative magnitude of E_1 and E_2. If there is an unequal power division in the hybrid or differences in the detector efficiencies the period of the output is 360 degrees but adjacent crossings or nulls will not be 180° apart. This can be corrected by using a dual null technique [34]. Adjacent nulls are obtained by adjustment of the calibrated phase shifter. The mid-point of the two settings of the phase shifter is considered to be the initial phase shifter setting. After a change in the unknown, two adjacent nulls and the mid-point between the required phase shifter settings are again obtained. The difference in the two mid-points is the phase shift of the unknown.

A phase bridge that is based upon the use of a balanced mixer and a complex differential null detector to take the difference between the audio voltages is commercially available. This system can achieve an accuracy of better than 1 degree over a 40 dB dynamic range of the unknown signal [34], [35].

Yu [36] developed a phase bridge that used a balanced mixer with somewhat different form of power combiner. The reference channel was split into two subchannels and one subchannel shifted 180 degrees with respect to the other by the use of delay lines. The unknown was then combined with each of these subchannels to form a sum and a difference output.

The interferometer or phase bridge with moveable probe may be used with a balanced mixer by combining the two channels in a slotted line which has two probes spaced a distance d apart as shown in Fig. 1(d). If we again assume signals in the two channels of the form of (1) and (2) and assume that the output from these probes is detected by balanced detectors and combined in a transformer or differential amplifier, the output of the selective amplifier can be shown to be

$$V \cong 4K_3 m \sin \beta d[E_1E_2 \sin(\phi \pm 2\beta L) - \theta] \cos \omega_m t \tag{9}$$

where K_3 is a constant. Again the E_1^2 and E_2^2 terms present in the unbalanced case have been cancelled and the output is similar to that of Fig. 1(c) except that the sensitivity is dependent upon the spacing d between probes. With a second detector synchronized with the modulating signal following the differential amplifier, both positive and negative outputs are available and hence the magnitude and sense of the phase difference is available. This system, which was proposed by Lacy [37], can be used as a moving probe system and the unknown phase shift interpreted in terms of the movement of the probes that is required to obtain a null. The system can also be made direct reading and a commercial phase meter is available using this technique [38].

Hamlin [39] recently proposed a dual probe system in which the output of the two probes is alternately switched to a single ended mixer. To balance the system the output of this mixer is minimized. Thus this is equivalent to taking the difference of the detected outputs of the two probes.

C. With Other Forms of Amplitude Modulation

In Section III-A we have assumed conventional amplitude modulation with both sidebands and the carrier present. If we now assume DSBSC modulation e_1 and e_2 are of the form

$$e_1 = E_1 m_1 \cos \omega_m t \cos(\omega t + \theta) \tag{10}$$

$$e_2 = E_2 m_2 \cos \omega_m t \cos(\omega t + \theta). \tag{11}$$

The output from a single ended mixer and selective amplifier with pass band centered at ω_m is zero. Although we will not consider it further here, as is true for most of these systems there is an output at 2ω which may be useful.

If we assume SSBWC modulation and e_1 and e_2 to be of the form

$$e_1 = E_1 \cos(\omega t + \theta) + \frac{E_{1m}}{2} \cos(\omega t + \omega_m t + \theta) \tag{12}$$

$$e_2 = E_2 \cos(\omega t + \phi) + \frac{E_{2m}}{2} \cos(\omega t + \omega_m t + \phi) \tag{13}$$

the output from a single-ended mixer and selective amplifier with pass band centered at ω_m is

$$V \cong Km\left[\frac{E_1^2}{2} + \frac{E_2^2}{2} + E_1E_2 \cos(\phi - \theta)\right] \cos \omega_m t \tag{14}$$

and from a balanced mixer and selective amplifier is

$$V \cong Km[2E_1E_2 \cos(\phi - \theta)] \cos \omega_m t. \tag{15}$$

Note that the output from a single ended mixer has nonlinear terms present which are again eliminated by the balanced mixer.

If we assume SSBSC modulation of e_1 and e_2 the output of the single-ended mixer at angular frequency ω_m is again zero as it was for the double sideband suppressed carrier modulation.

Although the relationship given in (15) is a useful output, and except for a constant is equivalent to that of (7), it was obtained by using single sideband modulation and hence a more complex system. As we will see later, the use of single sideband modulation in systems in which only one channel is modulated gives a very useful form of output and the added complexity required to obtain this type of modulation is worthwhile.

IV. Class I-b: CW Coherent Phase Bridge

If the signals in the two channels are unmodulated the output from a low pass filter and dc amplifier following the mixer in the systems of Fig. 1(a)–(d) would be, respectively

$$V \cong K_1 + K_2\left[\frac{E_1^2}{2} + \frac{E_2^2}{2} \pm E_1E_2 \cos(\phi - \theta)\right] \tag{16}$$

$$V \cong K_1 + K_2\left[\frac{E_1^2}{2} + \frac{E_2^2}{2} \pm E_1E_2 \cos(\phi \pm 2\beta L - \theta)\right] \tag{17}$$

$$V \cong K_1 + K_2[E_1E_2 \cos(\phi - \theta)] \tag{18}$$

$$V \cong K_1 + K_2[\sin \beta d][E_1E_2 \sin(\phi \pm 2\beta L - \theta] \tag{19}$$

where K_1 and K_2 are constants not the same for all cases and it is assumed that the difference between the outputs from the two detectors in the second and fourth case is taken in a differential dc amplifier. These equations for a dc output are of the same form as those discussed in Section III for an ac output at angular frequency ω_m and the comments in regard to the corresponding modulated systems apply.

Such systems are useful if it is preferable not to use modulation. On the other hand if the only requirement is that an unmodulated signal be applied to the component under test, it will be obvious that the systems in which the modulation is applied to only one channel can be used.

V. Class I-c: Coherent Phase Bridge with Single Channel Modulation

Possibly the first application of a modulated coherent detection system in which the two channels were not derived from a common modulated source, to the measurement of phase at microwave frequencies was made by Worthington [40]. He used SSBSC modulation although his technique has not been described in these terms. Robertson [41] proposed a system in which only one channel was modulated, using DSBSC modulation. Thus although the original systems used suppressed carrier modulation, we will take up systems in which only one channel is modulated, in the same order adopted for those systems which were based upon both channels being modulated.

A. DSBWC Modulated Systems

1) With Single-Ended Mixers: Phase measurement systems with one channel modulated with conventional amplitude modulation (DSBWC) and the channels combined in a single-ended mixer were proposed by Kido and Asai [42], Schafer [43], Garbacz and Eberle [44], Swarup and Yang [45], White [46], and King [47].

This system, identified as a "modulated subcarrier system" by Schafer, is shown in simplified form in Fig. 4. The CW source delivers a signal at angular frequency ω to each channel. The reference channel he termed the carrier channel and the channel with the signal under test, the subcarrier channel. The voltages impressed upon the mixer are of the form

$$e_1 = e_c = E_1 \cos(\omega t + \theta) \tag{20}$$

$$e_2 = e_{sc} = E_2(1 + m \cos \omega_m t) \cos (\omega t + \phi) \tag{21}$$

and it can be shown that the voltage from the detector at the modulation frequency will be of the form

$$V \cong Km[E_2^2 \pm E_1 E_2 \cos(\phi - \theta) \cos \omega_m t]. \tag{22}$$

If the combiner adds the two signals, a null in V occurs when

$$\cos(\phi - \theta) = \frac{E_2}{E_1} \cdot \tag{23}$$

Thus the phase of the reference signal (e_1) relative to that of the test signal (e_2) required for a null is

$$\phi = \theta \pm [180° - \Delta\phi] \tag{24}$$

where $\Delta\phi$ is always less than 90°. Schafer tabulated the angle at which a null occurs for various magnitude ratios of e_2 and e_1. For example, if the subcarrier is known to be at least 40 dB below the carrier at the initial and final settings, $88.43° \leq \Delta\phi < 90°$. However, for the test signal only 20 dB below the reference signal the possible error without correction increases to 5.7 degrees.

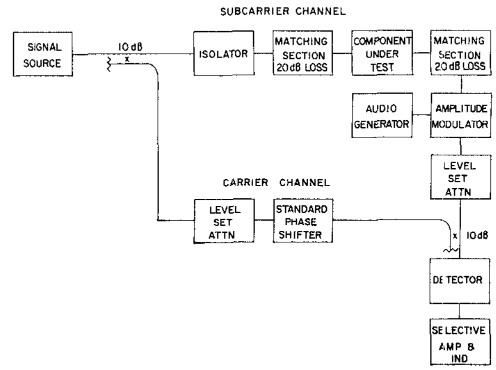


Fig. 4. Coherent phase bridge with one channel modulated (DSBWC) and single ended mixer (after Schafer).

To avoid having to measure the ratios of e_1 and e_2 and for precision measurements, Schafer proposed that a second null response be obtained, adjacent to the first, by adjusting the standard or reference phase shifter. The phase difference between test and reference signals will then be of the opposite sign. The average of these two readings of the standard is considered to be the corrected setting of the standard phase shifter. It is the reading which would produce a null if the two signals were of equal amplitude. This procedure also resolves the ambiguity of whether the reference leads or lags the test signal.

This system is in use at the National Bureau of Standards. If particular care is taken to account for the system errors it is capable of the maximum accuracy consistent with the state of the art.

2) With Balanced Mixer: Richmond [48] proposed a phase measurement system in which the signal in only one channel is modulated and in which the signals in the two channels are combined in a balanced mixer. This was an automatic system designed to measure the real and imaginary components of the unknown signal.

If (20) and (21) describe the signals incident upon a balanced mixer followed by a selective amplifier, the output at frequency ω_m will be

$$V \cong 2Km[E_1 E_2 \cos(\phi - \theta)] \cos \omega_m t. \tag{25}$$

The balanced mixer has eliminated the nonlinear E_2^2 term and the output is of the same form as (7). It should be pointed out that when a single-ended mixer is used and the signal to be measured is very small, the inherent noise modulated portion of the reference signal within the pass band of the selective amplifier may be comparable to the desired signal. Ideally the balanced mixer will cancel these noise modulation components.

An advantage of this system over the systems with both channels modulated is a relaxation of the requirement on the modulation frequency nulling circuit. If the reference signal is also modulated the measurement of small test signals may be interfered with by a residual output signal present due to imperfect nulling at the modulation frequency. For example, adjustment of the simple potentiometer R shown in Fig. 3 is usually sufficient to obtain a good null if a well balanced transformer and well matched detectors are used. The only undesired signal component produced by the nonlinear detectors due to the reference signal will be a dc signal which will be rejected by the transformer or selective amplifier. If the reference and test signals are both modulated a complex nulling circuit or an instrument such as the commercial differential null detector previously mentioned is usually necessary.

This system has also been described and used by Burton [49] and others.

B. DSBSC Modulated Systems

1) With Single-Ended Mixer: The system proposed by Robertson [41] and identified as a homodyne detection system utilizes DSBSC modulation in one channel. The voltages impressed upon the mixer become

$$e_1 = E_1 \cos(\omega t + \theta) \tag{26}$$

$$e_2 = E_2 \frac{m}{2} [\cos(\omega t + \omega_m t + \phi) + \cos(\omega t - \omega_m t + \phi)]. \tag{27}$$

Under our previous assumptions, the voltage from the selective amplifier will be

$$V \cong Km[E_1 E_2 \cos(\phi - \theta)] \cos \omega_m t. \tag{28}$$

The balanced modulation has suppressed the nonlinear terms just as the balanced mixer did. A null in V can be secured when

$$\phi - \theta = \pm n\pi/2 \quad (\text{where } n = 1, 3, 5, \cdots). \tag{29}$$

The requirement for a null is independent of the relative magnitude of E_1 or E_2. Thus the advantage of this system, as pointed out by Vernon [50], is that the measurement of phase can be made over an extremely wide dynamic range of the test signal E_2 without correction.

This advantage is never fully realized because an ideally balanced modulator does not exist. A residual carrier is always present and this carrier combines with the other signals to cause some error in the determination of $\phi - \theta$. Nevertheless, if a good balanced modulator is available the system is an improvement over the modulated subcarrier system unless the double null technique is used. Balanced modulators which are capable of supressing the carrier to a level 45 dB or more below the incident signal at the modulator have been reported. Thus if the incident signal at the detector from the unknown or test channel remains 20 dB or more below that from the reference channel, the magnitude ratio E_1/E_2 is increased to 65 dB or more. Using the tabulation by Schafer, a phase error due to the residual carrier should in this case be no more than approximately 0.04 degree. Systems based upon the DSBWC (modulated subcarrier) or DSBSC (homodyne) are being utilized by the NBS Electronic Calibration Center for offering relative phase calibration services on most passive microwave devices [51].

2) With Balanced Mixer: The use of balanced modulation in one of the channels and a balanced mixer results in an output from the selective amplifier of

$$V \cong 2Km[E_1 E_2 \cos(\phi - \theta)] \cos \omega_m t \tag{30}$$

which, except for a constant, is equivalent to (28). Sparks [52] used this system with a modulation frequency of 24 MHz and proposed that it be increased to 100 MHz for the measurement of very narrow RF pulses.

This circuit has been adopted for at least one commercial phase-meter in which the output of the balanced mixer controls the phase of the modulation voltage to the balanced modulator [53].

C. SSBWC Modulated Systems

1) With Single-Ended Mixer: If single sideband (with carrier present) modulation (SSBWC) is used in one channel the two signals presented to the detector are of the form

$$e_1 = E_1 \cos(\omega t + \theta) \tag{31}$$

$$e_2 = E_2 \cos(\omega t + \phi) + \frac{E_2 m}{2} \cos(\omega t \pm \omega_m t + \phi) \tag{32}$$

where the plus or minus sign indicates a choice between the upper or lower sideband. If these signals (assuming the upper sideband) are added and impressed upon a single-ended mixer the output at ω_m will be

$$V \cong \tfrac{1}{2} m[E_2{}^2 \cos \omega_m t + E_1 E_2 \cos[\omega_m t + (\phi - \theta)]]. \tag{33}$$

The mixing of a SSBWC signal with the coherent reference signal has transferred the phase information at RF frequency ω to the modulating frequency ω_m. This is a significant result. However, a null in V requires that

$$\cos[\omega_m t + (\phi - \theta)] = -\frac{E_2}{E_1} \cos \omega_m t. \tag{34}$$

This is possible if $E_1 = E_2$ and $(\phi - \theta) = \pi$.

This system has been used at UHF frequencies by Moore [54] who obtained the SSBWC modulation with electronic circuits. E_1 was made to be very much larger than E_2 and the first term was assumed to be negligible. Any contribution to V by the first term will introduce error.

2) With Balanced Mixer: If a balanced mixer is used the output from a selective amplifier will be

$$V \cong KmE_1 E_2 \cos[\omega_m t + (\phi - \theta)]. \tag{35}$$

The output at angular frequency ω_m now contains the phase information $(\phi - \theta)$, and a null in V is not dependent upon the ratio of the magnitudes of E_1 and E_2. If E_1 is held constant and the output measured with a phase insensitive indicator the relative amplitude of E_2 is available. If V and a reference signal at frequency f_m from the modulator are fed to an audio phase meter as shown in Fig. 5, the relative phase of e_2 with respect to e_1 will be directly indicated and may be recorded on an automatic system.

Mittra [56] used this measurement technique at X-band with a modulation frequency of 1000 Hz by using ferrite modulators. For example, SSBWC modulation can be obtained by separating the signal channel to be modulated into two RF signals which were 90° out of phase. These two signals are then modulated by two equal amplitude audio signals which are themselves 90° out of phase. If the two signals are then added in a hybrid, one of the sidebands will be suppressed [57]. Diode switch modulators that are presently available could be substituted for the ferrite modulators.

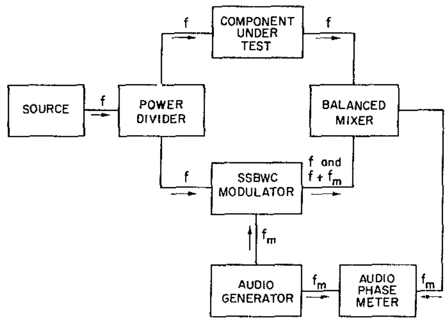


Fig. 5. Coherent phase bridge suitable for automatic read-out of relative phase.

D. SSBSC Modulated Systems

1) With Single-Ended Mixer: If one of the channels is modulated with a single sideband modulator and the carrier is suppressed, the two signals presented to the mixer are

$$e_1 = e_1 \cos(\omega t + \theta) \tag{36}$$

$$e_2 = \frac{E_2 m}{2} \cos(\omega t \pm \omega_m t + \phi). \tag{37}$$

Under the assumption that the upper sideband is used, the output from an unbalanced mixer is

$$V \cong \tfrac{1}{2} K m E_1 E_2 \cos[\omega_m t + \phi - \theta]. \tag{38}$$

Except for a constant this signal will be recognized as that from the SSBWC system and a balanced mixer.

Single sideband suppressed carrier signals have been obtained in many ways. Worthington [40] used a bridge with the reference channel very long compared to the measurement channel. By frequency modulating a klystron source with a 500 Hz sawtooth voltage he obtained a reference signal which varied continuously and linearly in phase with respect to the test signal. After combining the reference and test signals the output of the detector was displayed on an oscilloscope as a sine wave. Variation in phase of the unknown signal translated this sine wave along the sweep trace on the scope. Recording the movement of this wave was in effect comparing the phase of the detector output with the fixed phase of the scope sweep voltage.

Diemer and Knol [58] later showed that a continuous *linear* variation of phase of a signal will produce ideal frequency conversion or translation. A linear change in phase of 2π radians in a time t causes a frequency change of $2\,\pi/t$ Hz. Thus a continuous linear variation of phase of an RF signal will produce SSBSC modulation of that signal. Worthington had also suggested using a rotating vane phase shifter for frequency translation. Yaw [59] and Dropkin [60] developed systems based upon a motorized version of such a phase shifter using modulation frequencies of 210 Hz and 180 Hz, respectively.

SSBSC modulation can also be secured by employing stepped phase shifts to approximate a continuous or ideal sawtooth phase shift [61], [62], by filtering one sideband of a DSBSC signal [63], by using balanced modulation to suppress the carrier in the quadrature phasing method of securing SSBWC modulation which was referred to in Section V-C, [64], [65], or by using ferrite phase shifters [66]–[68].

Cumming [69], [70] used linear sawtooth modulation of the beam voltage of a traveling wave tube to produce frequency translation of microwave signals with a suppression of all other sideband band components, including the carrier, by at least 20 dB. This technique which consists of a linear sawtooth modulation of the transit time of an electronic device to produce frequency translation has been termed serrodyne frequency translation. Serrodyne frequency translation to secure SSBSC modulation has also been employed by Linker and Grimm [71] using 200 kHz modulation frequencies and by Goldbohm [72], Mathers [73], and Finnila et al. [74] using a 1 kHz modulation frequency. Israelsen and Haegele [75] used sinusoidal modulation of the electrodes of a traveling wave tube to secure a varying phase in one channel.

Although the rotary phase shifter method of obtaining frequency translation is possibly the simplest of the above methods, it is usually restricted to low modulation frequencies (a few hundred Hz) with a resultant increase in $1/f$ noise in the mixer and hence a reduction in sensitivity compared to that which can be obtained at higher modulation frequencies [76]. Systems that accomplish frequency translation by a sinusoidal modulating waveform should be used with caution since such systems may have an output spectrum that is symmetrical, with undesired components present. If possible, these components should be removed by a filter. If the carrier is not sufficiently suppressed the system must be analyzed in terms of the SSBWC systems previously discussed.

2) With Balanced Mixer: If a balanced mixer is used the output at ω_m becomes

$$V \cong K m E_1 E_2 \cos[\omega_m t + (\phi - \theta)] \tag{39}$$

which except for a constant is equivalent to (38). A complex automatic phase measuring system was developed by Mullen and Carlson [55] using this basic circuit. Linker and Grimm [77] reported on a symmetrical three channel X-band bridge which used 20 kHz serrodyne modulation and balanced mixers as shown in simplified form in Fig. 6. The symmetry of this balanced system tends to improve the accuracy of phase measurements as a function of frequency.

Ernst [78] also used SSBSC modulation and a balanced mixer in a system designed to measure phase modulation or rapid changes in phase at 70 GHz. The signal from the mixer was fed to a wide-band limiter-discriminator with center frequency f_m. For small changes, the output voltage is directly proportioned to the rate of phase change of the unknown signal.

VI. Class II: Phase Bridge with Frequency Conversion

The signals in the two channels of the phase bridge may be translated to a new frequency by heterodyning these signals with a separate local oscillator as shown in Fig. 7. For a local oscillator signal e_1 at angular frequency ω_1 and a test signal e_{2s} at ω_2, the signals applied to each mixer are of the form

$$e_1 = E_1 \cos(\omega_1 t + \alpha) \tag{40}$$

$$e_{2s} = E_{2s} \cos(\omega_2 t + \phi). \tag{41}$$

The second-order components of the mixing process are

$$\begin{aligned} V_S \cong \frac{E_1^2}{2} + \frac{E_{2s}^2}{2} + \frac{E_1^2}{2} \cos 2(\omega_1 t + \alpha) \\ + \frac{E_{2s}^2}{2} \cos 2(\omega_2 t + \phi) \\ + E_1 E_{2s} \cos(\omega_1 t + \omega_2 t + \phi + \alpha) \\ + E_1 E_{2s} \cos(\omega_2 t - \omega_1 t + \phi - \alpha). \end{aligned} \tag{42}$$

Only the last term is of interest since the amplifiers following the mixer are normally tuned to reject all components except those centered at the intermediate frequency $\omega_2 - \omega_1$. The relative phase difference $(\phi - \alpha)$ between the RF signals has been transferred to the constant frequency IF output. Similarly if the signal in the reference channel is

$$e_{2R} = E_{2R} \cos(\omega_2 t + \theta) \tag{43}$$

the output from the mixer in the reference channel at angular frequency $\omega_2 - \omega_1$ would be

$$V_R \cong E_1 E_{2R} \cos(\omega_2 t - \omega_1 t + \theta - \alpha). \tag{44}$$

These two signals V_R and V_S which have been translated to frequency $\omega_2 - \omega_1$ may be amplified and compared in a low-frequency phase meter.

It is obvious that this heterodyne process could be repeated to a still lower frequency and that many different final frequencies have been chosen for comparison. For example, Ring [21] used a single conversion to one

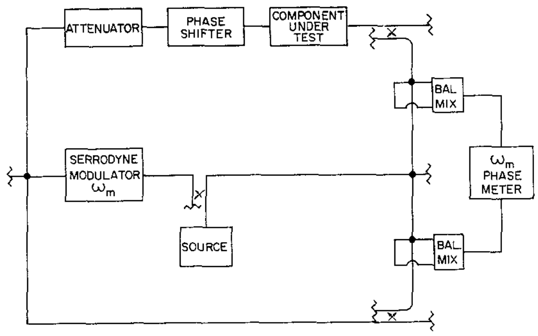


Fig. 6. Symmetrical three channel phase bridge (after Linker and Grimm).

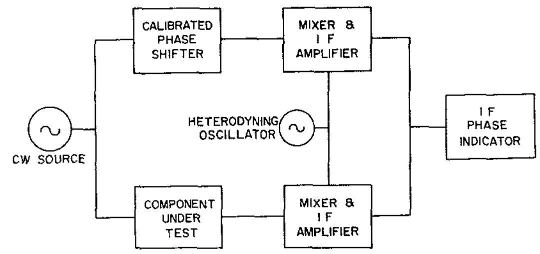


Fig. 7. Use of dual mixers with a common local oscillator for translation of RF phase information to an intermediate frequency.

MHz in a phase measurement system; Barrett and Barnes [79] used a 30 MHz IF for an automatic phase plotter.

Goodman [80] used dual conversion to 15 Hz in an automatic impedance plotter. Stevens [81] used dual conversion down to 2.78 kHz and then used the zero crossings of this audio signal to control the counting of pulses from a 10 MHz clock. Operating between 100 to 500 MHz he estimated an accuracy of measurement of the phase between two signals to within 0.2 degree for CW signals and to within 0.5 degree for pulsed RF. Chamberlain et al. [82] used a 60 MHz intermediate frequency. Koontz [83] used multiple conversion down to 37 MHz for a system designed to measure dispersed pulse transmitter systems. Blore et al. [84] used a phase-locked local oscillator and source with conversion to 60 mHz and then to 100 kHz to measure components at 35 and 70 GHz in a complex system.

It is possible to make these systems relatively insensitive to changes in the amplitude of the unknown signal by using IF amplification and matched phase-stable amplitude limiters at the IF frequency [79], [85].

One commercial phase meter is based upon this principle [86]. It uses a separate local oscillator locked to an A.F.C. circuit to follow variations in the frequency of the source. Over a frequency range of 300 to 2400 MHz the signals in the two channels of the bridge are heterodyned to an IF frequency between 10 and 11 MHz. These signals are then amplified and limited in amplitude and applied to the ends of an artificial line used as a combiner. The amplitude limitation of the two IF channels insures E_1 being equal to E_2 and hence the conditions of (7) apply.

VII. Combinations of Class I and II Systems

Sophisticated systems are possible using various combinations of the above classes. For example, Clayton et al. [87] developed a phase and amplitude system for use over a range of frequencies from 20 MHz to over 40 GHz by translating all frequencies to 65 MHz in each channel. This involved double conversion over the 20 MHz to 1.95 GHz range. The reference channel at 65 MHz was then SSBSC modulated with $f_m = 146$ Hz and the output of the mixer fed to an audio phase comparator for direct readout or recording. This system is available as a commercial item [88].

VIII. Systems for Direct Readout of Relative Phase

Direct readout of phase information is possible by several schemes. It has already been pointed out that a balanced mixer is a phase discriminator which can be used to furnish an error signal to drive various forms of electronic or servo-controlled reference phase shifters to renull the phase bridge. The variation of the phase reference can, after calibration, provide an analog or digital readout of relative phase. A direct readout technique for the dual probe system has also been discussed. Very convenient direct reading systems can be assembled using the SSB modulation techniques in which the RF phase information is transferred to the modulation frequency. If the modulation frequency is in the low RF or audio range direct reading phase meters are available which provide an indication of phase on digital or conventional meters and an analog output to drive an oscilloscope or recorder.

Another interesting form of display has been proposed. It was recognized by Tucker [3] that the output from the filtered mixer of a coherent detection system is proportional to the cosine of the phase angle between the signals in the two channels. Cosgriff [89] and others pointed out that this output was proportional to the real component of the unknown signal and that if the phase of one or the other of the signals in the two channels was shifted $\pi/2$ radians the output would be proportional to the imaginary component of the unknown signal. Both of these components can be made available simultaneously with the circuit shown in Fig. 8. The signals e_1 and e_2 in the two channels are each split into equal in-phase components e_{1a} and e_{1b}, and e_{2a} and e_{2b}, by hybrids A and B. Two of the split channels, say e_{1a} and e_{2a} are combined in hybrid C and a balanced mixer to give an output proportional to $\cos(\phi-\theta)$. The signal in one of the other split channels, say e_{2b}, is shifted 90° in phase and then combined with e_{1b} in hybrid D and a balanced mixer to give an output proportional to $\sin(\phi-\theta)$. These outputs may then be applied to the deflection plates of an oscilloscope. The deflection of the trace from the center of the face is proportional to the relative magnitude of the test signal and the angular orientation of this spot on the face of the tube is proportional to the phase angle of the test signal. Samuel [10] proposed this form of display in 1947 for automatically plotting the complex reflection coefficient plane on an oscilloscope and pointed out that this was a coherent detection system. For convenience and symmetry he placed 45 degree phase shifts in each of two of the split channels to get an effective total shift of 90 degrees. This merely rotates the orthogonal axis of the reflection coefficient plane by 45 degrees relative to the axis of the deflection plates. This circuit was also used by Bachman [90] for an automatic reflection coefficient plotter.

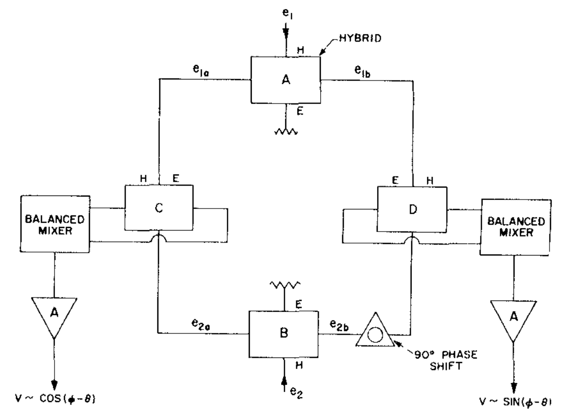


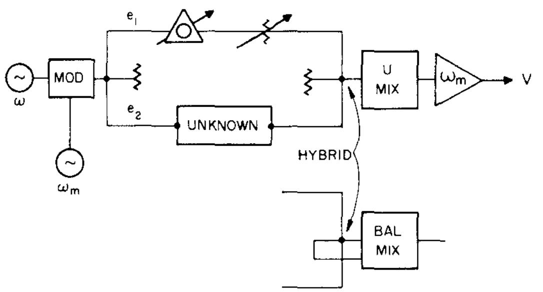
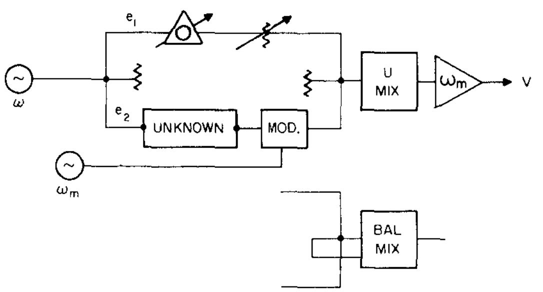
Fig. 8. Phase discriminator circuit that provides outputs proportional to the sine and cosine of the relative phase angle of the unknown signal. A 3 dB coupler may be used in place of hybrid B and the 90° phase shifter.

In a novel adaptation of this principle Cohn and Oltman [91] proposed a similar network for an automatic swept frequency phase measurement system. In this system the split of the test signal was obtained with a 3 dB coupler which simultaneously gives the required 90 degree phase shift. They also used single-ended mixers which were sufficient since their phase bridge used DSBSC modulation in one channel. The outputs of the two mixers were applied to a commercial ratio meter. Since this instrument takes the ratio of the two input signals, it reads directly the tangent of $(\phi-\theta)$. This circuit is used in a commercially available system [92].

Strandberg [93], and Kaiser et al. [94] also used this circuit with balanced mixers and applied the output of the two mixers to the horizontal and vertical deflection amplifiers of a scope to display the complex reflection coefficient or transmission coefficient plane.

This hybrid network of two separate mixers forms the basis for two different commercially available phase measurement systems, and excellent discussions of the characteristics of this type of output, the system errors involved, and the individual modifications of this basic system are available [95], [96].

Griffin [97] used an adaption of this circuit for the measurement of pulsed signals. The reference signal was derived from a stable local oscillator and the cathode ray tube trace was blanked at regular intervals by clock controlled oscillator. Thus the angular distance between successive points on the display is proportional to the average frequency difference over the given time interval.

SIMPLIFIED CIRCUIT	COMMON SYSTEM INDENTIFICATION	AMPLITUDE MODULATION		V = VOLTAGE OUTPUT AT FREQUENCY OF MODULATION (ω_m)	
			TYPE	FROM UNBALANCED MIXER(5)	FROM BALANCED MIXER
	PHASE BRIDGE	BOTH CHANNELS COHERENTLY MODULATED	DSBWC	$K\left[E_1^2+E_2^2 \pm 2E_1E_2\cos\phi\right]\cos\omega_m t$	$4K\left[E_1E_2\cos\phi\right]\cos\omega_m t$
			DSBSC	(SEE NOTE 6)	———
			SSBWC	$K\left[\frac{E_1^2}{2}+\frac{E_2^2}{2}\pm E_1E_2\cos\phi\right]\cos\omega_m t$	$2K\left[E_1E_2\cos\phi\right]\cos\omega_m t$
			SSBSC	———	———
	MODULATED SUB-CARRIER	ONLY CHANNEL CARRYING e_2 MODULATED (SEE NOTE 4)	DSBWC	$K\left[E_2^2 \pm E_1E_2\cos\phi\right]\cos\omega_m t$	$2K\left[E_1E_2\cos\phi\right]\cos\omega_m t$
	HOMODYNE		DSBSC	$K\left[\pm E_1E_2\cos\phi\right]\cos\omega_m t$	$2K\left[E_1E_2\cos\phi\right]\cos\omega_m t$
	———		SSBWC	$K\left[\frac{E_2^2}{2}\cos\omega_m t \pm \frac{E_1E_2}{2}\cos(\omega_m t+\phi)\right]$	$KE_1E_2\cos(\omega_m t+\phi)$
	SERRODYNE; FREQUENCY TRANSLATION		SSBSC	$K\left[\pm \frac{E_1E_2}{2}\cos(\omega_m t+\phi)\right]$	$KE_1E_2\cos(\omega_m t+\phi)$

NOTES

1. R-F VOLTAGES $e_1 = E_1\cos\omega t$, AND $e_2 = E_2\cos(\omega t+\phi)$ ARE MODULATED AT ANGULAR FREQUENCY ω_m.
2. ϕ IS THE RELATIVE PHASE BETWEEN R-F VOLTAGES e_1 AND e_2 AT FREQUENCY ω.
3. K IS A CONSTANT NOT NECESSARILY THE SAME IN ALL CASES.
4. IF ONLY CHANNEL CARRYING e_1 IS MODULATED INTERCHANGE E_1 AND E_2 IN EXPRESSION FOR OUTPUT.
5. ADDITION OR SUBTRACTION OF e_1 AND e_2 BEFORE APPLICATION TO MIXER DETERMINES SIGN OF ± TERM IN OUTPUT OF UNBALANCED MIXER
6. WITH BOTH CHANNELS COHERENTLY MODULATED THE CARRIER MUST BE PRESENT TO PRODUCE A FILTERED OUTPUT AT ω_m.

Fig. 9. Output from coherent phase measurement bridge as a function of the type of modulation and mixer employed.

IX. General Considerations

A convenient summary of the classification of the basic phase measurement systems is shown in Fig. 9. This clearly points out the relationship between these systems.

The various systems have been shown in simplified form for analytic purposes. Specific details of the systems are available in the cited references. For reasons of space, an analysis of the various sources of error cannot be considered here. There are, however, several excellent papers which should be consulted before serious measurements are undertaken. For example, Beatty [98] has expressed the basic phase shift relationships of a two-port under test in terms of the reflection coefficients of the two-port and the system in which it is inserted. Hunton [99], Lacy [100], Leed [101], and Kuhn [102] have considered the analysis of microwave measurement components and systems by means of signal flow graphs. Schafer [103] considered in detail general mismatch errors in phase measurements. Phillips [8] published several nomographs useful in the determination of phase uncertainties due to mismatched components. Schafer and Beatty [104] have made a thorough error analysis of a standard microwave phase shifter, and Augustine and Cheal [105] have considered the errors in two broadband phase shifters due to mismatched terminations.

Errors in specific types of systems have been considered by several authors. An excellent discussion of the system errors involved in the simple phase bridge is available in a paper by Magid [19]. Ishii and Schumacher [106] have also made an analysis of the simple phase bridge. A discussion and allocation of errors in a phase bridge with balanced mixer is also available [34]. Ellerbruch [51] has made a critical evaluation of the modulated subcarrier and homodyne systems, and Mathers [73] and Rubin [107] have considered in detail the errors in serrodyne systems.

Evaluation of the capabilities of the various systems should be made in the light of what can be done with them with presently available components. Many of these systems were developed ten or fifteen years ago and the estimates of achievable measurement accuracy made at that time can be improved upon today.

The backward diode is a relatively new mixer element that should find wide use in these systems. The backward diode is a form of tunnel diode in which the tunneling process is restricted and the negative resistance region virtually disappears. The abrupt, alloyed junction, and high doping, yield extremely low diode output noise in the audio range [108]. At one kHz these diodes have been measured to be up to 35 dB quieter than the 1N23WE and 25 dB quieter than the 1N1838, a point contact diode designed for low-frequency IF systems [109]. This marked decrease in $1/f$ noise makes the backward diode very attractive for the coherent detection systems [110]–[112]. These diodes have received attention in the literature for use in Doppler radar systems. They should be attractive for phase measurement systems. To take full advantage of the characteristics of these diodes they should be used with low-noise, low-impedance amplifiers. These amplifiers are appearing on the market.

We have been concerned with phase measurement systems as such. Some of these circuits find wide use in the instrumentation for other research areas, as for example in spectroscopy and radio astronomy. Some of the circuits were developed independently in these research areas. To keep the number of references within manageable proportions this literature has not been covered.

In the classification of the systems, mention was made of several commercially available systems. This was done to help in the identification of the types of circuits. It is not intended to be an all-inclusive listing of commercial phase meters available for the UHF and microwave frequencies. We have been concerned with identification of the basic circuit. There are many, sometimes subtle, modifications that can be made to these circuits that can affect the convenience of operation, accuracy and resolution, frequency coverage, packaging and cost. The choice of a commercial system must take all these factors into account.

X. Summary

Methods of measuring relative phase at UHF and microwave frequencies which depend upon a two (or more) channel bridge circuit for comparison of an unknown and reference signal can be classified in terms of the modulation applied to the signals in the individual channels and the type of mixer used to combine these channels. When so analyzed the intimate relationship between all of these systems, and many of the advantages and disadvantages of the various systems, become obvious.

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